3.2 Rational Equations MATH 1610 THOMPSON

1) Solve the equation.

$$\frac{3}{2x-3} = \frac{2}{x+5}$$

cross multiply

$2(2x - 3) = 3(x + 5)$

$4x - 6 = 3x + 15$

$x = \mathbf{21}$

2) Solve the following equation involving rational expressions. Then, identify the equation as an identity, an inconsistent equation, or a conditional equation.

$$\frac{z+4}{z-7} = \frac{10}{-7}$$

cross multiply

$10(z - 7) = -7(z + 4)$

$10z - 70 = -7z - 28$

$17z = 42$

$x = \mathbf{\frac{42}{17}}$

a conditional equation

A conditional equation has at least one real answer
Identity equals itself with all real numbers
An inconsistent equation is an equation that has no solution.

3) Solve the equation.

$$\frac{7}{2x-5} = \frac{4}{x+1}$$

cross multiply

$4(2x - 5) = 7(x + 1)$

$8x - 20 = 7x + 7$

$x = \mathbf{27}$

4) Find the real solutions, if any, of the following equation. Use the quadratic formula.

$$\cancel{X(x-1)}\ \frac{2x}{\cancel{x-1}} + \cancel{X}(x-1)\ \frac{1}{\cancel{x}} = 3\ X(x-1)$$

Multiply all by x(x-1)

$2x^2 + x - 1 = 3x(x-1)$ distribute

$2x^2 + x - 1 = 3x^2 - 3x$ move all to the right

$0 = x^2 - 4x + 1$ use quadratic equation

$$\frac{4 \pm \sqrt{16-4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{12}}{2}$$

$\sqrt{4 \cdot 3}$

$$\frac{4 \pm 2\sqrt{3}}{2} =$$

$$\mathbf{2 \pm \sqrt{3}}$$

5) Find the real solutions, if any, of the following equation. Use the quadratic formula.

x x-1 x(x-1)

$$\frac{5x}{\cancel{x-1}} + \frac{2}{\cancel{x}} = 6$$

Multiply all by x(x-1)

$5x^2 + 2x - 2 = 6x(x-1)$ distribute

$5x^2 + 2x - 2 = 6x^2 - 6x$ move all to the right

$0 = x^2 - 8x + 2$ use quadratic equation

$\sqrt{4 \cdot 14}$

$$\frac{8 \pm \sqrt{64 - 4(1)(2)}}{2(1)} = \frac{4 \pm \sqrt{56}}{2}$$

$$\frac{4 \pm 2\sqrt{14}}{2} =$$

$$\mathbf{2 \pm \sqrt{14}}$$

6) Solve the following equation by factoring.

$$45x - 56 = \frac{45}{x}$$

Multiply all by x

$45x^2 - 56x = 45$ move 45 to the left

$45x^2 - 56x - 45 = 0$ factor

$(9x + 5)(5x - 9) = 0$

$9x+5=0$ $5x-9=0$

$$-\frac{5}{9}, \frac{9}{5}$$

DOUBLE	HALF
1	45
3	15
5	9

7) Solve the following equation by factoring.

$$20x - 9 = \frac{20}{x}$$

Multiply all by x

$20x^2 - 9x = 20$ move 20 to the left

$20x^2 - 9x - 20 = 0$ factor

$(5x + 4)(4x - 5) = 0$

$5x+4=0$ $4x-5=0$

$$-\frac{4}{5}, \frac{5}{4}$$

8) Solve the equation.

$$\frac{x^2-6x+9}{x+5}=0$$

Solve the top **only** by factoring

$x^2 - 6x + 9$

$(x-3)(x-3)$

3

9) Solve the equation.

$$\frac{x^2-2x+1}{x+4}=0$$

Solve the top **only** by factoring

$x^2 - 6x + 9$

$(x-3)(x-3)$

3

10) Solve the equation by making an appropriate substitution.

$$\left(x-\frac{18}{x}\right)^2-4\left(x-\frac{18}{x}\right)-21=0$$

u is always the middle term

Let u $x-\frac{18}{x}$ then the quadratic equation in u is $u^2-4u-21=0$

$(u-7)(u+3)=0$

$u = 7, -3$

$x-\frac{18}{x}$ = 7 and $x-\frac{18}{x}$ = -3 multiply all by x

$x^2 - 18 = 7x$ and $x^2 - 18 = -3x$

$x^2 - 7x - 18 = 0$ and $x^2 + 3x - 18 = 0$

$(x-9)(x+2)=0$ and $(x-3)(x+6)=0$

the solution set is **9, -2, 3, -6**

11) Solve the equation by making an appropriate substitution.

$$\left(x-\frac{30}{x}\right)^2-6\left(x-\frac{30}{x}\right)-7=0$$

u is always the middle term

Let u $x-\frac{30}{x}$ then the quadratic equation in u is $u^2-6u-7=0$

$(u-7)(u+1)=0$

$u = 7, -1$

$x-\frac{30}{x} = 7$ and $x-\frac{30}{x} = -1$ **multiply all by x**

$x^2 - 30 = 7x$ and $x^2 - 30 = -x$

$x^2 - 7x - 30 = 0$ and $x^2 + x - 30 = 0$

$(x-10)(x+3)=0$ and $(x-5)(x+6)=0$

the solution set is **10, -3, 5, -6**

12) $\frac{1}{(x+4)^2} = \frac{1}{x+4} + 56$ Use $u=\frac{1}{x+4}$

$u^2=u+56=0$

$u^2-u-56=0$

$(u+7)(u-8)=0$ $u = -7,8$

$\frac{1}{x+4} = -7$ and $\frac{1}{x+4} = 8$

$-7x-28=1$ $8x+32= 1$

$-7x= 29$ $8x = -31$

$x = \frac{29}{7}, \frac{-31}{8}$

13) $\frac{1}{(x+8)^2} = \frac{2}{x+8} + 48$ Use $u=\frac{1}{x+8}$

$u^2=2u+48=0$

$u^2-2u-48=0$

$(u+6)(u-8)=0$ $u = -6, 8$

$\frac{1}{x+8} = -6$ and $\frac{1}{x+8} = 8$

$-6x-48=1$ $8x+64=1$

$x = \frac{49}{6}, \frac{-63}{8}$

14) $x^{-2} - 6x^{-1} + 8 = 0$ Use $u= x^{-1}$

$u^2-6u+8=0$

$(u-2)(u-4)=0$ $u = 2,4$

$x^{-1} = 2$ and $x^{-1} =4$

$x = \frac{1}{2}, \frac{1}{4}$

15) $x^{-2} - 9x^{-1} + 20 = 0$ Use $u= x^{-1}$

$u^2-9u+20=0$

$(u-5)(u-4)=0$ $u = 5,4$

$x^{-1} = 5$ and $x^{-1} =4$

$x = \frac{1}{5}, \frac{1}{4}$

16) $\left(\frac{v}{v+2}\right)^2 - \frac{3v}{v+2} = 10$

$\left(\frac{v}{v+2}\right)^2 - 3\frac{\text{v}}{\text{v+2}} = 10$ Use u=$\frac{v}{v+2}$

u^2- 3u=10

u^2-3u-10=0

(u-5)(u+2)=0 u = 5,-2

$\frac{v}{v+2}$ = 5 and $\frac{v}{v+2}$ = -2

5v + 10 = v -2v – 4 = v

4v = -10 -3v = 4

x = $-\frac{10}{4} = -\frac{5}{2}$, $-\frac{4}{3}$

17) $\left(\frac{v}{v+2}\right)^2 + \frac{v}{v+2} = 12$

$\left(\frac{v}{v+2}\right)^2 + \frac{\text{v}}{\text{v+2}} = 12$ Use u=$\frac{v}{v+2}$

u^2 + u=12

u^2+ u-12=0

(u-3)(u+4)=0 u = 3,-4

$\frac{v}{v+2}$ = 3 and $\frac{v}{v+2}$ = -4

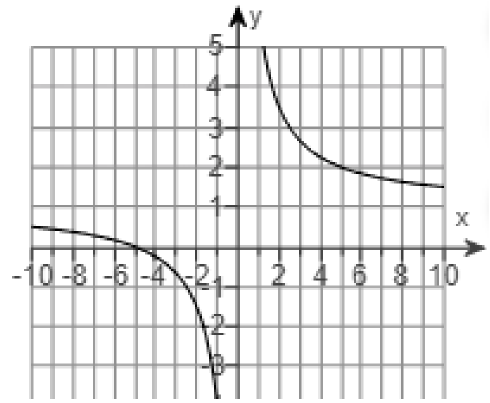
3v + 6 = v -4v – 8 = v

2v = - 6 -5v = 8

x =$-\mathbf{3}$, $-\frac{8}{5}$

18) Two possible solutions to the equation $f(x) = 0$ are listed. Use the given graph of $y = f(x)$ to decide which, if any, are extraneous.

$x = -5$ or $x = -4$



Graph crosses at x= -5 but not x= - 4

Select the correct choice below and, if necessary, fill in the answer box within your choice.

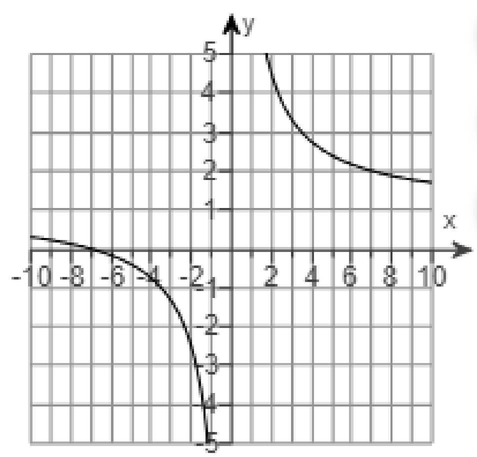
A. The extraneous solution(s) is/are $x = -4$.
(Use a comma to separate answers as needed. Type integers or fractions.)

B. There is no extraneous solution.

Solutions that are not x-intercepts of the graph of f(x)=0 are called extraneous solutions.

19) Two possible solutions to the equation $f(x) = 0$ are listed. Use the given graph of $y = f(x)$ to decide which, if any, are extraneous.

$x = -7$ or $x = -4$



Graph crosses at x= -7 but not x= - 4

Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. The extraneous solution(s) is/are $x = -4$.
(Use a comma to separate answers as needed. Type integers or fractions.)

B. There is no extraneous solution.

Solutions that are not x-intercepts of the graph of f(x)=0 are called extraneous solutions.